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Computational Complexity in Analysis

SoSe 2015, Exercise Sheet #3

EXERCISE 5:

a) Prove that the following decision problem can be solved in exponential, but not in polynomial, time:

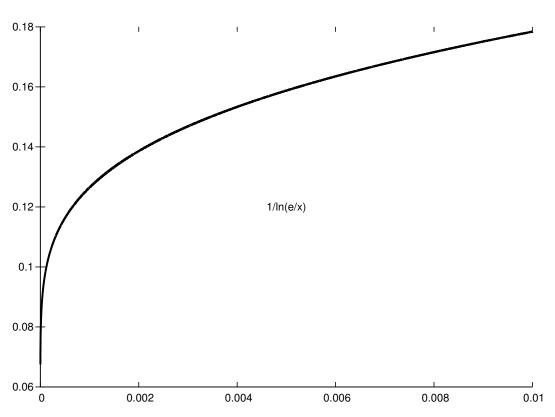
 $\{ \mathbb{1}^k \cap \langle \mathcal{M} \rangle \mid k \in \mathbb{N}, \text{ Turing machine } \mathcal{M} \text{ rejects input } \mathbb{1}^k \cap \langle \mathcal{M} \rangle, \}$

or takes more than $k \cdot |\langle \mathcal{M} \rangle|^k$ steps to accept it $\}$

b) $L \subseteq \mathbb{N}$ is decidable in polynomial time iff the real number $\sum_{x \in L} 4^{-x}$ is polytime computable.

EXERCISE 6:

- a) Prove exp: $[-2^k;k] \to \mathbb{R}$ computable within parameterized polynomial time $(n+k)^{O(1)}$.
- b) Prove that exp : $\mathbb{R} \to \mathbb{R}$ is not computable within time t(n) for any $t : \mathbb{N} \to \mathbb{N}$.
- c) Prove that the following function $h: [0;1] \rightarrow [0;1]$ is computable in exponential, but not in polynomial time:



$$h(0) = 0,$$
 $h(t) = 1/\ln(e/t)$