Martin Ziegler
Sewon Park, GyuHyeon Choi, Junsung Lim, Won-young Lee

Issued on May 03, 2016
Solutions due: May 10, 2016

CS500

Spring 2016, Assignment #3

Recall that a Boolean term Φ in 4-conjunctive normal form (4CNF) looks like this:

$$(\ell_1 \vee \ell_2 \vee \ell_3 \vee \ell_4) \wedge (\ell_5 \vee \ell_6 \vee \ell_7 \vee \ell_8) \wedge \ldots \wedge (\ell_{4m-3} \vee \ell_{4m-2} \vee \ell_{4m-1} \vee \ell_{4m}) \ , \qquad (1)$$

where each ℓ_j is a *literal*, that is, some variable x_i or its negation $\neg x_i$.

PROBLEM 10 (7+3P) **:**
Establish that Φ from Equation (1) in variables $x_1, \ldots, x_n$ admits a satisfying assignment iff the following $\widetilde{\Phi}$ in 3CNF and variables $x_1, \ldots, x_n, y_1, \ldots, y_m$ does:

$$\widetilde{\Phi} \quad = \quad (\ell_1 \vee \ell_2 \vee y_1) \wedge (\neg y_1 \vee \ell_3 \vee \ell_4) \ \wedge \ (\ell_5 \vee \ell_6 \vee y_2) \wedge (\neg y_2 \vee \ell_7 \vee \ell_8) \ \wedge \\ \wedge \ldots \wedge \ (\ell_{4m-3} \vee \ell_{4m-2} \vee y_m) \wedge (\neg y_m \vee \ell_{4m-1} \vee \ell_{4m})$$

Argue that (an encoding of) $\widetilde{\Phi}$ can be computed from (an encoding of) Φ in time polynomial in the length of the input.

PROBLEM 11 (2+6+2P)**:**
Fix a term $\widetilde{\Phi}$ in 3CNF with m clauses $(\ell_{3j-2} \vee \ell_{3j-1} \vee \ell_{3j})$ in variables $x_1, \ldots, x_n$. Now consider the graph $G_{\widetilde{\Phi}}$ consisting of vertex set V and undirected edges E, where

$$V \quad = \quad \big\{(j,1),(j,2),(j,3) : 1 \leq j \leq m\big\}, \quad E \quad = \quad \big\{\{(j,s),(i,t)\} : j = i \vee \ell_{3j+1-s} = \neg \ell_{3i+1-t}\big\}$$

a) Draw the graph $G_{\widetilde{\Phi}}$ corresponding to

$$\widetilde{\Phi} \quad = \quad (\neg x_1 \vee x_2 \vee \neg x_3) \wedge (x_1 \vee \neg x_2 \vee x_3) \wedge (x_1 \vee x_2 \vee x_3)$$

b) Establish for every 3CNF $\widetilde{\Phi}$ that $\widetilde{\Phi}$ admits a satisfying assignment iff $G_{\widetilde{\Phi}}$ contains $\geq m$ pairwise non-adjacent vertices.

c) Argue that (an encoding of) $G_{\widetilde{\Phi}}$ can be computed from (an encoding of) $\widetilde{\Phi}$ in time polynomial in the length of the input.