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Issued on May 19, 2016
Solutions due: May 31, 2016

CS500

Spring 2016, Assignment #4

PROBLEM 12 (6+4P) **:**

a) Which of the following structures satisfy the first-order formula "$\forall x\, \exists y.\, x+y=0$", and why not?

i) $(\mathbb{N},+,0)$
ii) $(\mathbb{Z},+,0)$
iii) $(\{0,1\}^d,\mathrm{XOR},(0,\ldots 0))$
iv) $(\mathbb{Z},\cdot,0)$
v) $(\mathbb{Z},\cdot,1)$
vi) $(\mathbb{Q},\cdot,1)$
vii) $(\mathbb{Q}\setminus\{0\},\cdot,1)$
viii) $(\mathbb{C}\setminus\{0\},\cdot,1)$
ix) $(\mathbb{R}^3\setminus\{0\},\times,(1,1,1))$
x) $\left(\mathbb{Q}^{2\times 2}\setminus\{0\},\cdot,\left(\begin{smallmatrix}1&0\\0&1\end{smallmatrix}\right)\right)$
xi) $\left(\left\{\left(\begin{smallmatrix}a&b\\c&d\end{smallmatrix}\right): a,b,c,d\in\mathbb{Q}, ad\neq bc\right\},\cdot,\left(\begin{smallmatrix}1&0\\0&1\end{smallmatrix}\right)\right)$
xii) $(\{f:X\to X \text{ injective}\},\circ,\mathrm{id})$, for which sets X ?

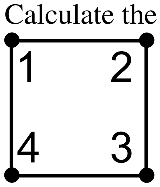
b) Consider a Boolean term $\varphi\equiv C_1\wedge C_2\wedge\cdots\wedge C_m$ in CNF over variables $x_1,\ldots,x_n$.
Encode it as the logical structure (U,V,N,P) with

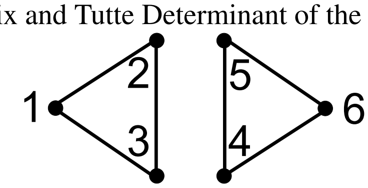
- universe $U=\{0,1,\ldots,n+m-1\}$ (representing variables and clauses),
- unary relation $V=\{0,1,\ldots,n-1\}$ (representing variables),
- binary relation $N=\{(v,c):$ variable v occurs negated in clause $c\}$, and
- binary relation $P=\{(v,c):$ variable v occurs unnegated in clause $c\}$.

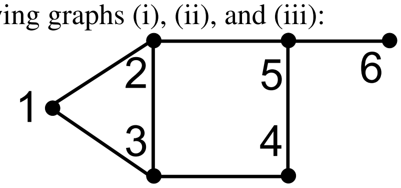
Moreover consider a subset $T\subseteq U$ as an assignment of value `true` to variables $V\cap T$ and value `false` to variables $V\setminus T$. Now construct a first-order formula $\Phi(T,V,N,P)$ expressing whether T constitutes a satisfying assignment to φ.

PROBLEM 13 (6+4P) **:**

a) Calculate the Tutte Matrix and Tutte Determinant of the following graphs (i), (ii), and (iii):







b) Estimate the asymptotic number of terms in expanded form of the n-variate polynomial $\prod_{j=1}^{n}(X_j+1)$.

PROBLEM 14 (4+4+2P)**:**

a) Suppose $\mathcal{A}$ is a randomized algorithm solving the decision problem L in time $t(n)$ with one-sided error $1-\varepsilon$ independent of n: On inputs $\vec{x}\notin L$, $\mathcal{A}$ always correctly reports `false`; but on inputs $\vec{x}\in L$, $\mathcal{A}$ might also report `false` with probability $1-\varepsilon$. Design and analyze an algorithm $\mathcal{A}'$ that, by repeating $\mathcal{A}$ an appropriate (which?) number of times, errs only with probability $2^{-100\cdot n}$.

b) Now suppose randomized $\mathcal{B}$ makes two-sided errors: reporting `false` on inputs $\vec{x}\in L$ with probability $\leq 1/2-\varepsilon$ and `true` on inputs $\vec{x}\notin L$ with probability $\leq 1/2-\varepsilon$ for some fixed $\varepsilon>0$. Design and analyze an algorithm $\mathcal{B}'$ that repeatedly invokes $\mathcal{B}$ to err with probability $\leq 2^{-100\cdot n}$.

c) Search, select, study, and summarize two publications on causes and rates of so-called *soft errors*.