Algorithmic Foundations of Numerics

Schedule: Tue.+Thu. 14h30—15h45 in E3 #3445

Instructor: Martin Ziegler, **TA:** 박지원

Language: English **Prereq.s:** Analysis + TCS

Attendance: 10 points for missing <5 lectures, 9 when missing 5, 8 when missing 6, and so on.

Grading: Homework 15%, Presentation 15%, Midterm 30%, Final exam 30%, Attendance 10%

Homework: Assigned roughly every 2nd week, 7 days to solve, individual solutions by email

Literature, slides, assignments etc:
`http://theoryofcomputation.asia/16CS700/`

Exams: Midterm Oct 20, Final Dec 15, 1pm-3pm

Overview

1. Recap on discrete Theory of Computation:
   * computability theory
   * asymptotic runtime/memory
   * machine models
   * P, NP_1, NP, #P, PSPACE, EXP
   * reduction, completeness
   * parameterized complexity
2. Real Computability Theory:
   * numbers, sequences, limits
   * uncomputable equality test
   * computability vs. continuity
   * arithmetic and operators
   * uncomputable Wave equation
   * enrichment, analytic functions
   * multivaluedness, linear algebra
3. Computability on Metric Spaces
   * C[0;1] and Weierstrass
   * compact Euclidean subsets
   * representations and TTE
   * Kreitz/Weihrauch "Main Thm"
   * Henkin-continuity
   * Weihrauch reduction
4. Complexity Theory over Reals
   * complexity and continuity
   * maximizing polytime functions
   * integration and solving ODEs
   * solving Poisson's PDE
   * parametrized analytic functions
5. Imperative Real Programming
6. Complexity on Metric Spaces

Background Check

- Convergent sequence
- Continuous function
- Compact subset
- Metric space
- Logic?
- C++
- Unix/Linux

- Halting Problem
- Models of Computation
- Oracle machine
- $\mathcal{NP}$, reduction
- Approx. algorithm

Theoretical Computer Science *and* Mathematics

"***Virtues***":

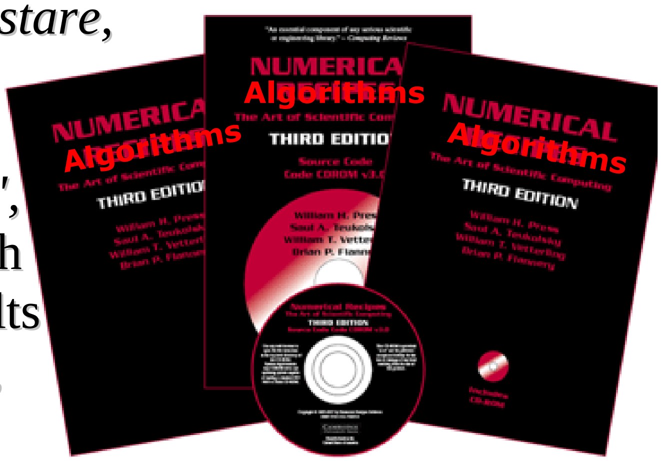
- problem specification
- formal semantics
- algorithm design
- and analysis (correctness, efficiency)
- optimality proof

Reliability in Numerical Software?

Peter Linz (Courant Institute), p.412, Bull. AMS vol.19:2

«Over the years, I have sat on many Ph.D. qualifying examinations or dissertation defenses for engineering students whose work involved a significant amount of numerical computing. In one form or another, I invariably ask […]: "How do you know that your answers are as accurate as you claim?" *[…]*

After an initial blank or hostile stare, I usually get an answer like "I tested the method with some simple examples and it worked", "I repeated the computation with several values of *n* and the results agreed to three decimal places", *or more lamely*, "the answers looked like what I expected".



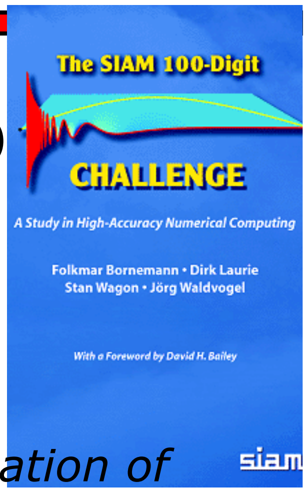
Specification in Numerics?

nag_opt_one_var_deriv (e04bbc) *normally* computes a sequence of x values which *tend in the limit* to a minimum of F x subject to the given bounds

use and pray

- NAG Numerical Library, >1700 routines
- MATLAB, • GNU Scientific Library

- W.Tucker (2002): "*Rigorous ODE Solver and Smale's 14th Problem*" (Lorenz Attractor)
- SIAM 100 Digits Challenge (2004)
- T.Hales (Fulkerson Prize 2009): numerical proof of Kepler's conjecture
- D.J. Platt (2013): "*Numerical Computations Concerning the GRH*" and "*Numerical Verification of the Ternary Goldbach Conjecture*" (with H.A.Helfgott).



Debunking Numerical Myths

Must not test for equality "**=**"

How about inequality "**<**" ?

$x=0 \iff \neg(x<0) \wedge \neg(x>0)$

→ *multivalued* semantics

Orevkov'63
Jockusch &Soare'72

[Specker'59] There is a computable C^∞ $f:[0;1]\to[0;1]$ attaining its minimum in no computable point

[Pour-El&Richards'89]
There is a computable initial condition f s.t. solution $u(1)$ is not computable (contains encoding of Halting problem)

$$\Delta u = \ddot{u}$$
$$u(0) = f$$
$$u(0)=0$$

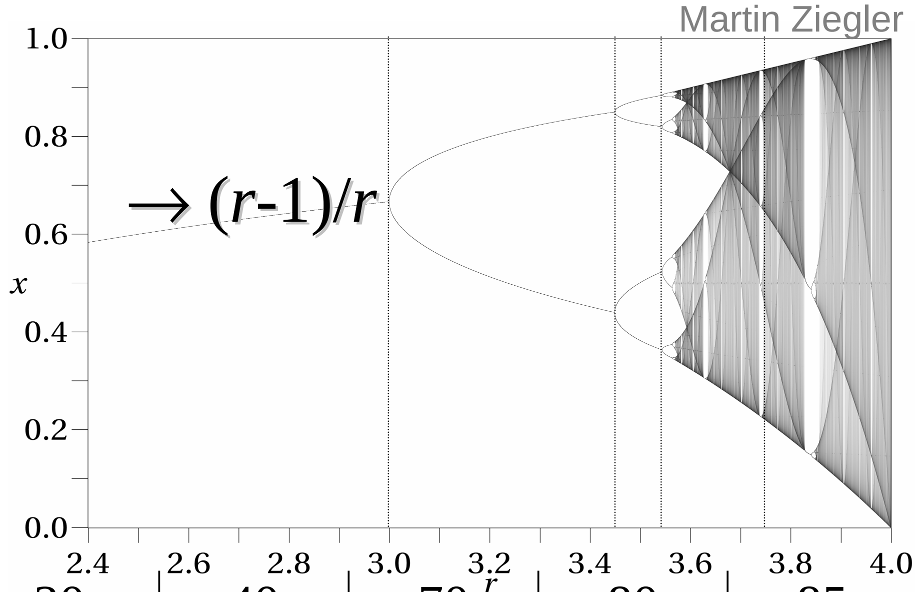
Weihrauch&Zhong: "*Is Wave Propagation Computable or Can Wave Computers Beat the Turing Machine?*", Proc. London Math. Soc.'02

Iterating the Logistic Map

Martin Ziegler

$$x_n \to x_{n+1} := r\cdot x_n\cdot(1-x_n),$$
$$1<r<4$$

$r=15/4$,
$x_0=½$



$n=$	25	30	40	70	80	85
float	0.81505	0.71524	0.84782			
double	0.81494	0.71810	0.41635	0.45263	0.87641	0.55053
long double	0.81494	0.71810	0.41635	0.45220	0.85619	0.81688