Martin Ziegler
Jiwon Park

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CS700

Fall 2016, Assignment #2

PROBLEM 2 (1+2+3+1+3P)**:**

a) Let $f :\subseteq \mathbb{R} \to \mathbb{R}$ be a computable partial function and $(r_j) \subseteq \mathrm{dom}(f)$ a computable sequence. Prove that the sequence $f(r_j)$ is again computable.

b) Let $(s_j) \subseteq \mathbb{R} \setminus \{0\}$ be a computable sequence.
Prove that the sequence $\mathrm{sign}(s_j) \in \{-1, +1\}$ is computable.

c) Let $f : [0;1] \to [-1;1]$ be a computable function with $f(0) < 0 < f(1)$. Prove that f has a computable root.
Hint: In case f has no dyadic rational root, apply (a) and (b) to the sequence (r_j) of dyadic rationals in $[0;1]$.

d) Let $g : [0;1] \to [0;1]$ be a computable function.
Prove that it has a computable fixed point, that is, some $x \in [0;1]$ with $g(x) = x$.

e) Suppose compact $K \subseteq \mathbb{R}^d$ has non-empty interior and let $h : K \to K$ be a (componentwise) computable function which is c-Lipschitz for some $c < 1$. Prove that h has a (componentwise) computable fixed point.